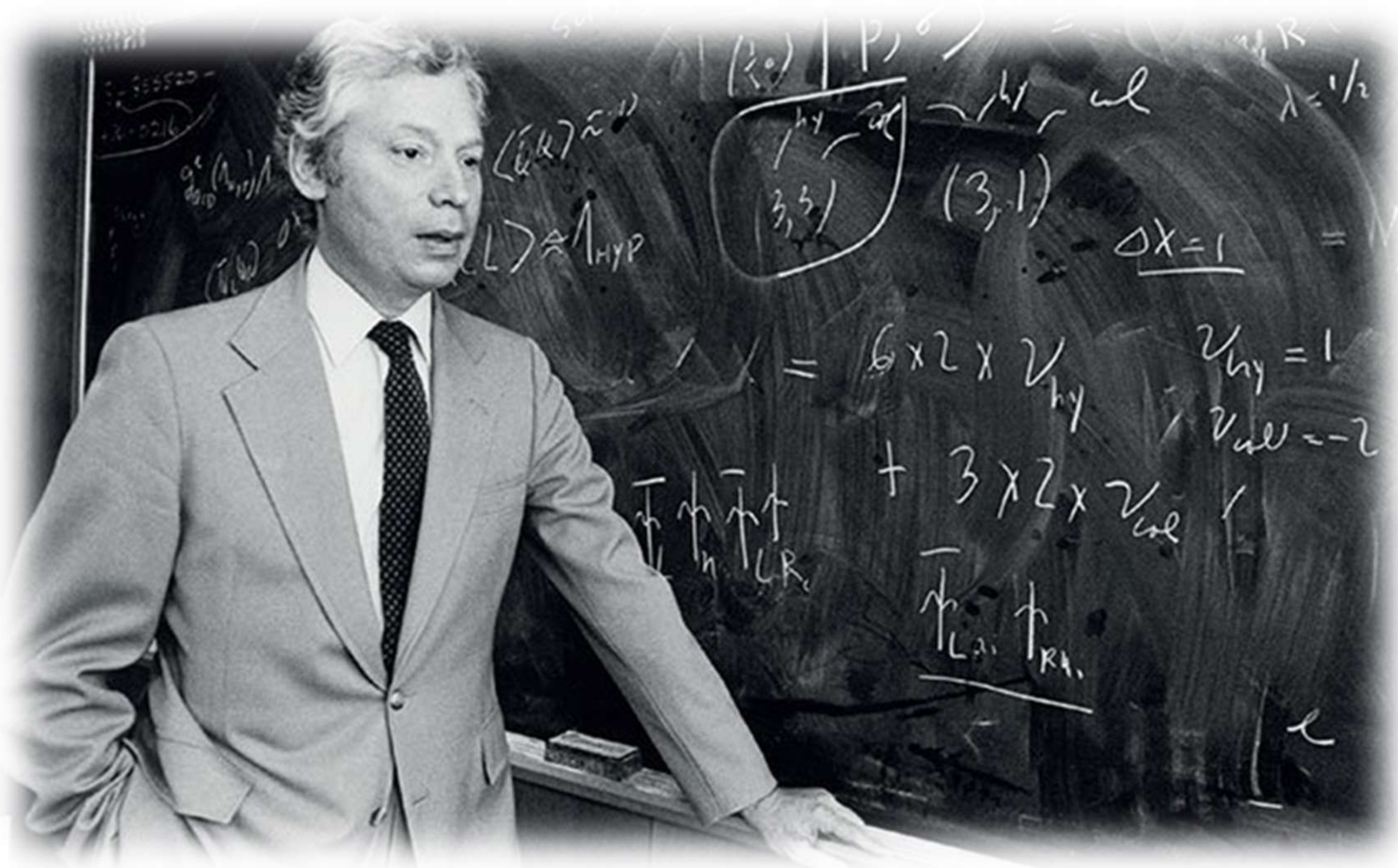
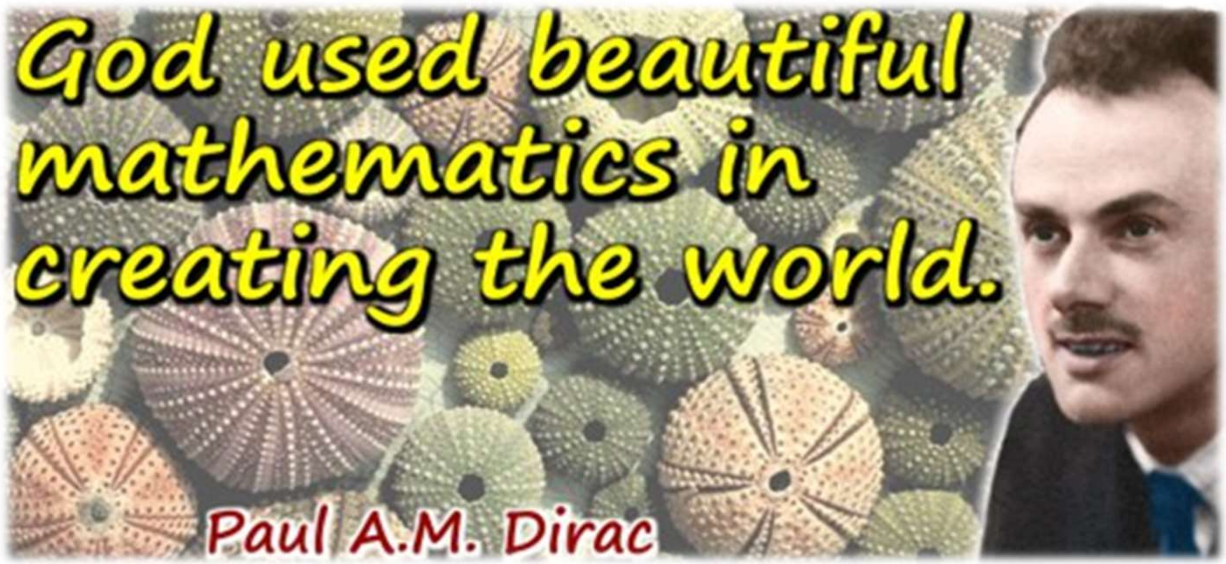
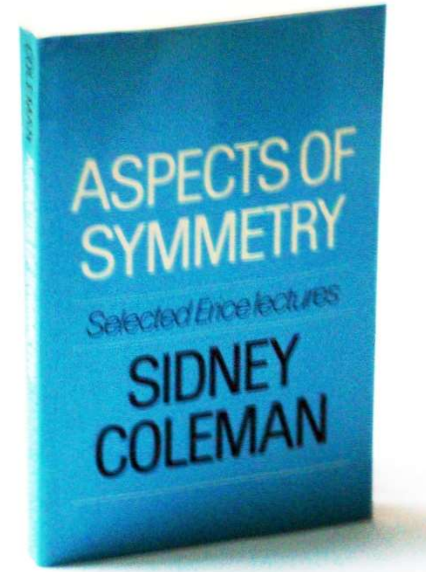


“Nothing in physics seems so hopeful to as the idea that it is possible for a theory to have a high degree of symmetry was hidden from us in everyday life. The physicist’s task is to find this deeper symmetry.”



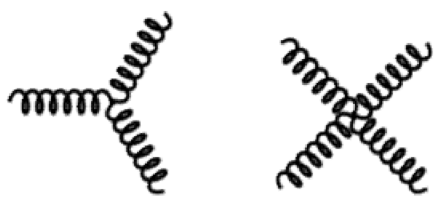


"In the simplest array of digits he [Ramanujan] detected wonderful properties: congruences, symmetries and relationships which had escaped the notice of even the outstandingly gifted theoreticians."
-- James R. Newman

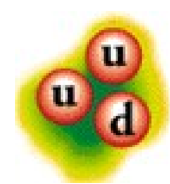
Quantum ChromoDynamics

$$\frac{1}{4g^2}\mathrm{tr}F^2 + \overline{\psi}\,\not{D}\psi$$

interactions



matter



G

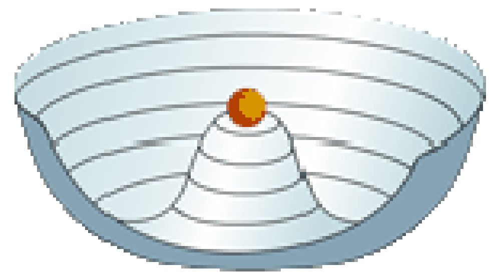
Lie group

R

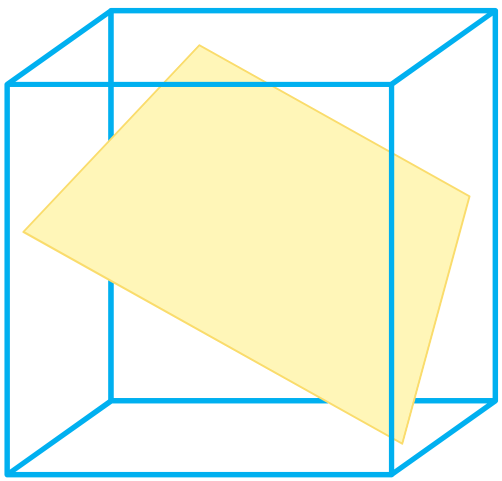
representation

N_f copies

$$Gr(k,N) = \mathbb{C}^{kN} /\!/ \, U(k)$$



$$Gr(k, N) = \frac{U(N)}{U(k) \times U(N-k)}$$



Theorem:

Frobenius algebra → 2d TQFT

Theorem:

Modular Tensor Category → 3d TQFT

Moore-Seiberg, Witten, Reshetikhin-Turaev

M_4 = ALE space

Moduli problem:

$$F_A^+ + \ldots = 0$$

$$\sum_n q^n \, \chi\left(\mathcal{M}_n^{\mathrm{inst}}(M_4)\right) \; = \; \chi_{\mathrm{VOA}}(q)$$

[K.Yoshioka]
[H.Nakajima]
[C.Vafa, E.Witten]
:

Topology

Algebra

3-manifolds → Modular Tensor Categories

4-manifolds → (mock & quantum) modular forms

VOA characters

Topological Modular Forms

Topology	Physics	Algebra
		Modular Tensor Categories
3-manifolds →	3d N=2 →	
		(mock & quantum) modular forms
4-manifolds →	2d (0,2) SQCD →	
		VOA characters
		Topological Modular Forms

Topology | Physics | Algebra

3+3 = 6

3-manifolds → 3d N=2 →

4-manifolds → 2d (0,2) SQCD →

4+2 = 6

Modular Tensor Categories

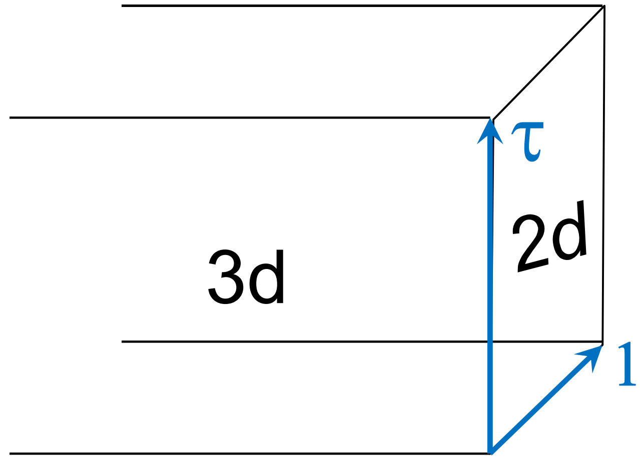
(mock & quantum) modular forms

VOA characters

Topological Modular Forms

Topology	Physics	Algebra
		Modular Tensor Categories
3-manifolds	3d N=2	(mock & quantum) modular forms
4-manifolds	2d (0,2) SQCD	VOA characters
		Topological Modular Forms

Today

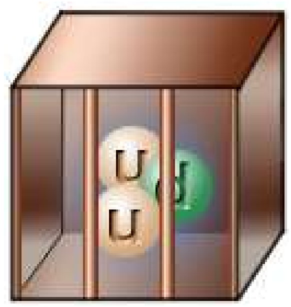


w/ A.Gadde, P.Putrov (2013)

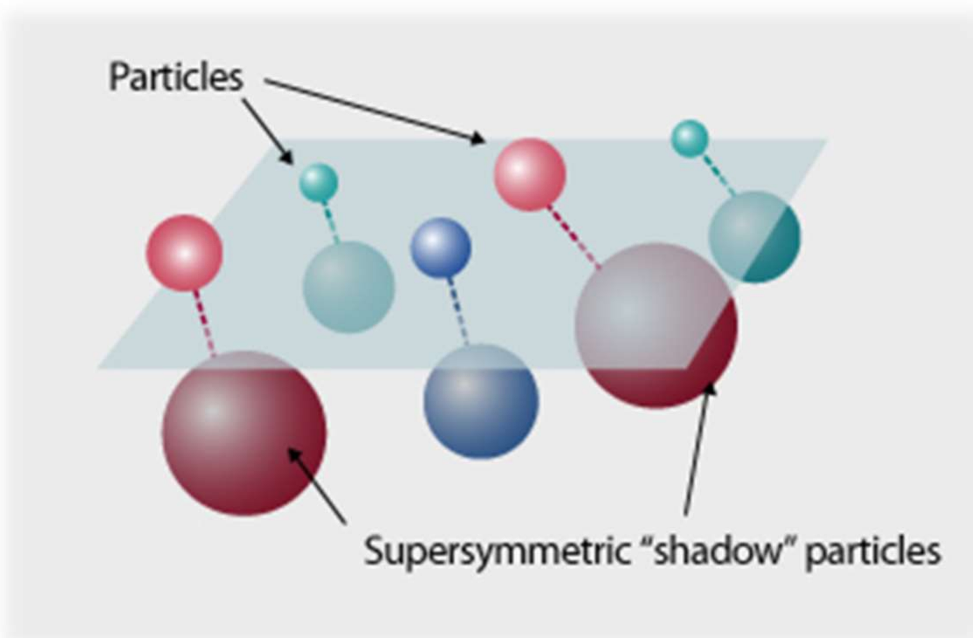
1) Half-index
= 2d/3d elliptic genus

2) 2d (0,2) SQCD

$$\frac{1}{4g^2}\,\mathrm{tr}\,F^2 + \overline{\psi}\,\not{D}\psi$$



$$\frac{1}{4g^2}\mathrm{tr}\,F^2 + \overline{\psi}\,\not{D}\psi$$

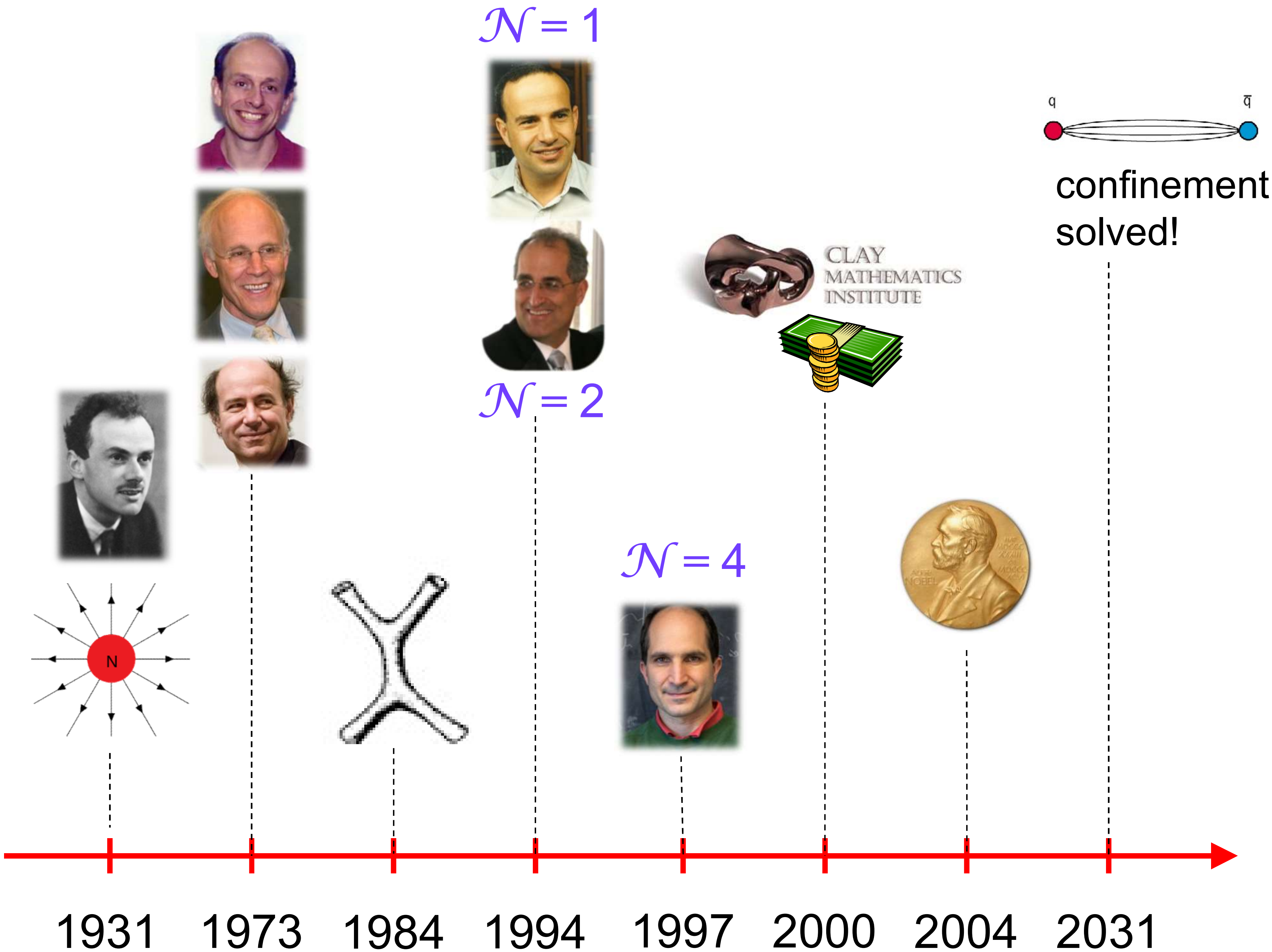


$\mathcal{N} = 1$

$\mathcal{N} = 2$

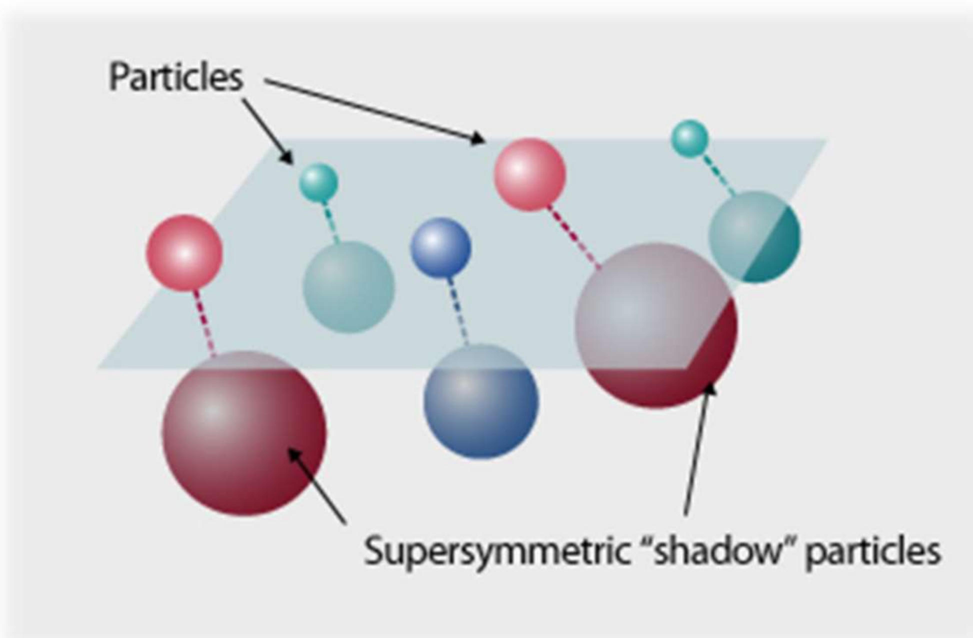
$\mathcal{N} = 3$

$\mathcal{N} = 4$



$\mathcal{N} = 1$
$\mathcal{N} = 2$
$\mathcal{N} = 4$
N
CLAY
MATHEMATICS
INSTITUTE
q
$\bar{q}$
confinement solved!
1931
1973
1984
1994
1997
2000
2004
2031

$$\int d^{2}x\,\frac{1}{4g^2}\mathrm{tr}F^2 + \overline{\psi}\,\not{D}\psi$$



$\mathcal{N} = (0,1)$

$\mathcal{N} = (0,2)$

$\mathcal{N} = (2,2)$

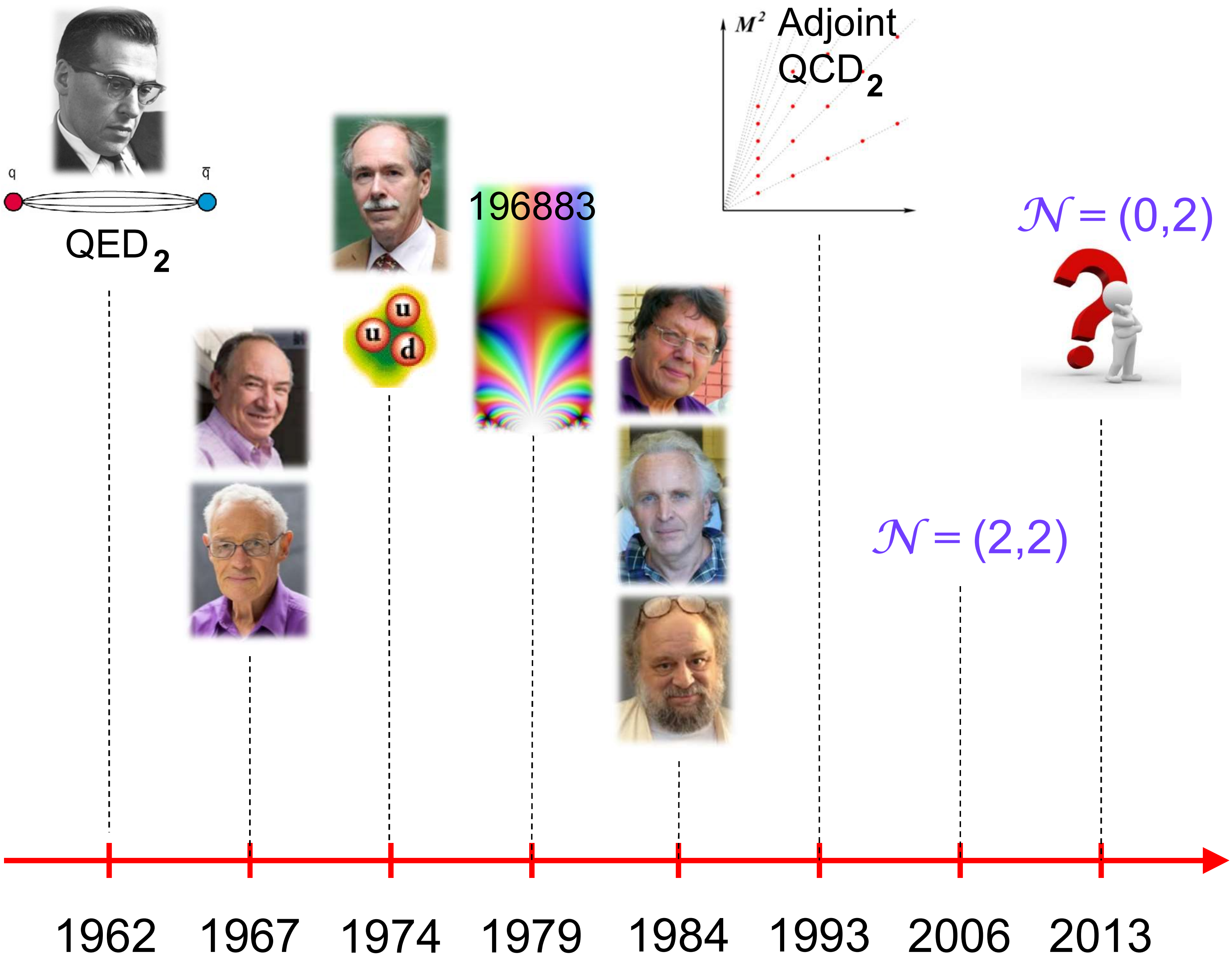
⋮

4d $\mathcal{N} = 1$

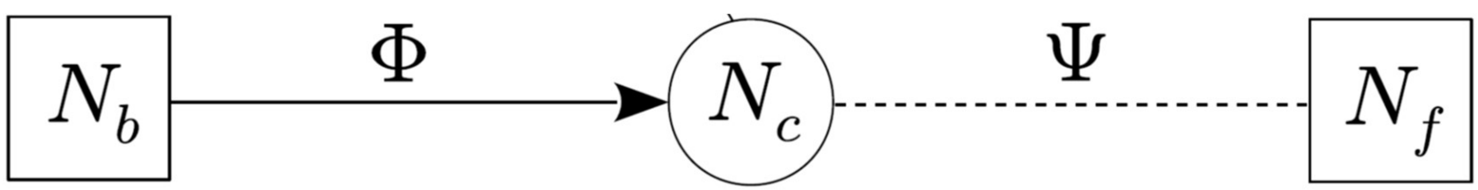
4d $\mathcal{N} = 2$

2d $\mathcal{N} = (0,2)$

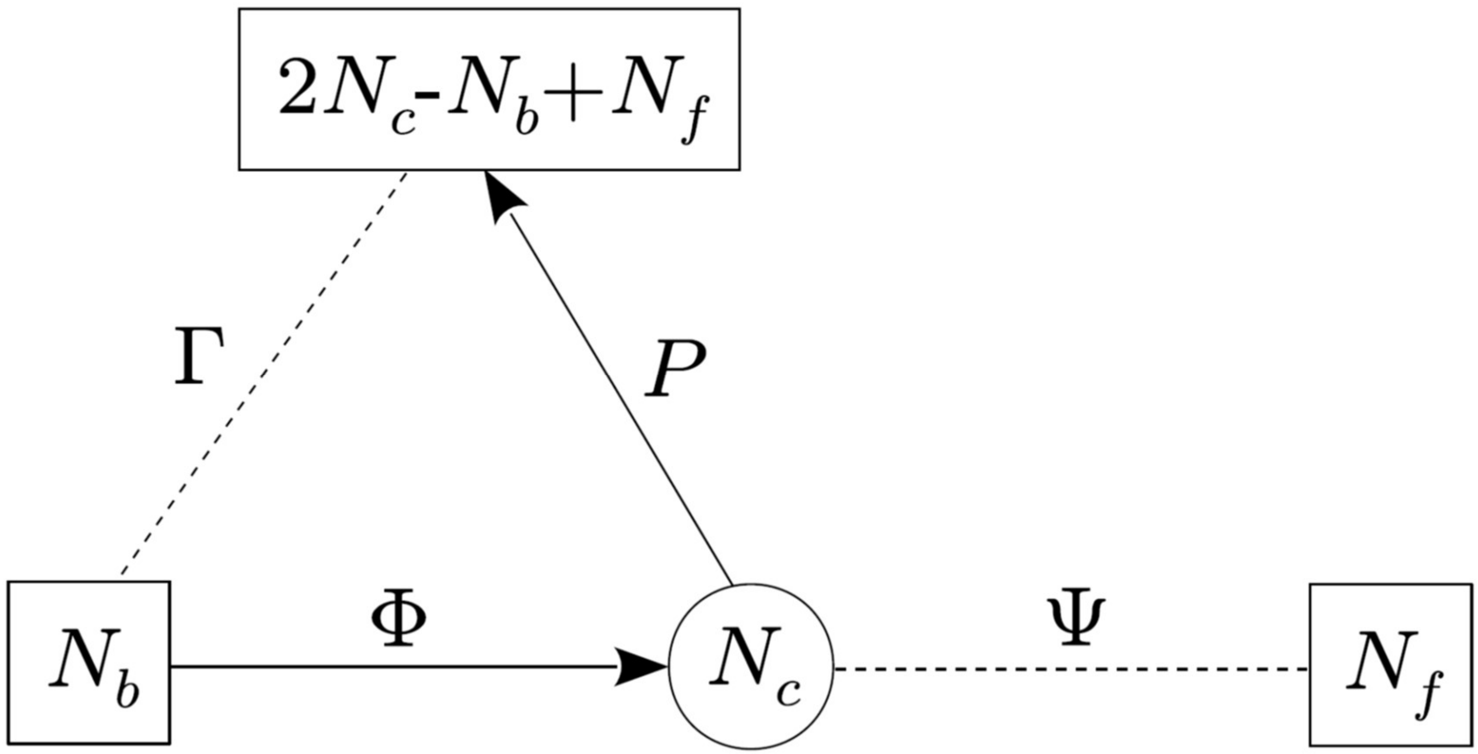
2d $\mathcal{N} = (2,2)$



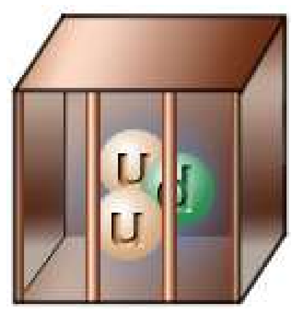
q
q̄
QED2
196883
M²
Adjoint QCD2
𝒩 = (0,2)
𝒩 = (2,2)
1962
1967
1974
1979
1984
1993
2006
2013



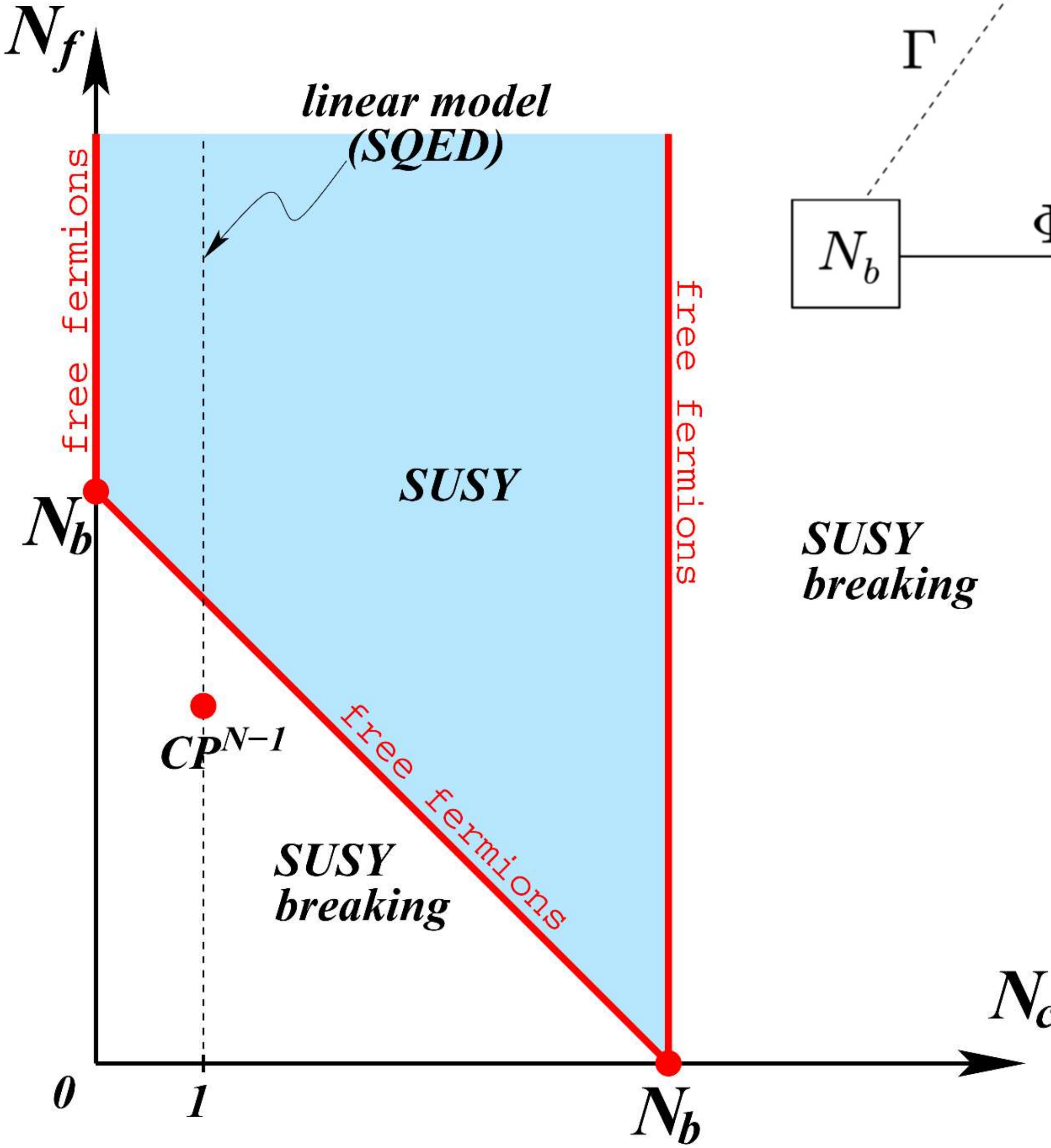
N_b
Φ
N_c
Ψ
N_f

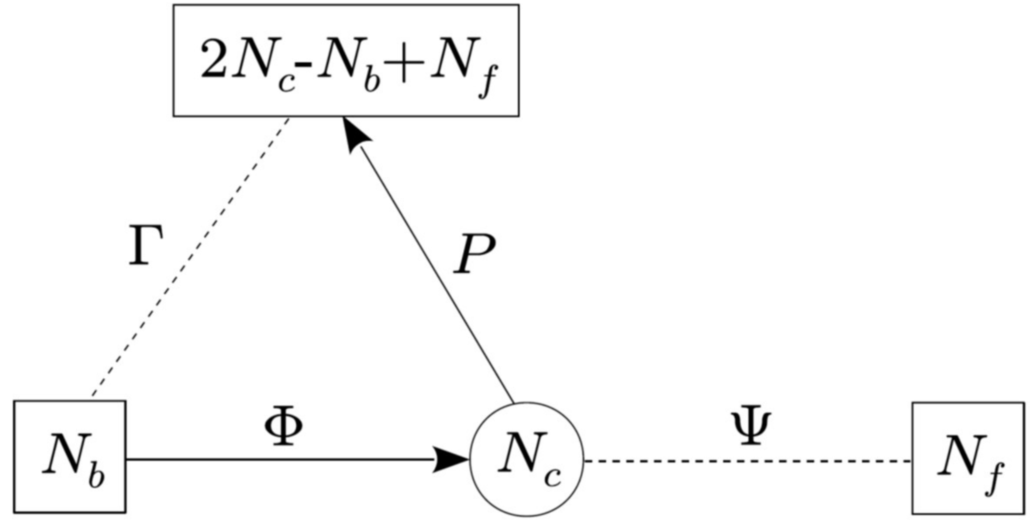


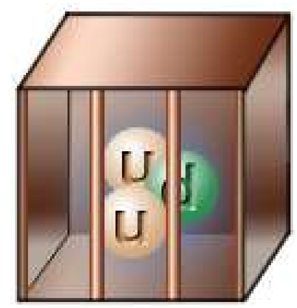
$2N_c$-N_b+N_f
Γ
P
N_b
Φ
N_c
Ψ
N_f



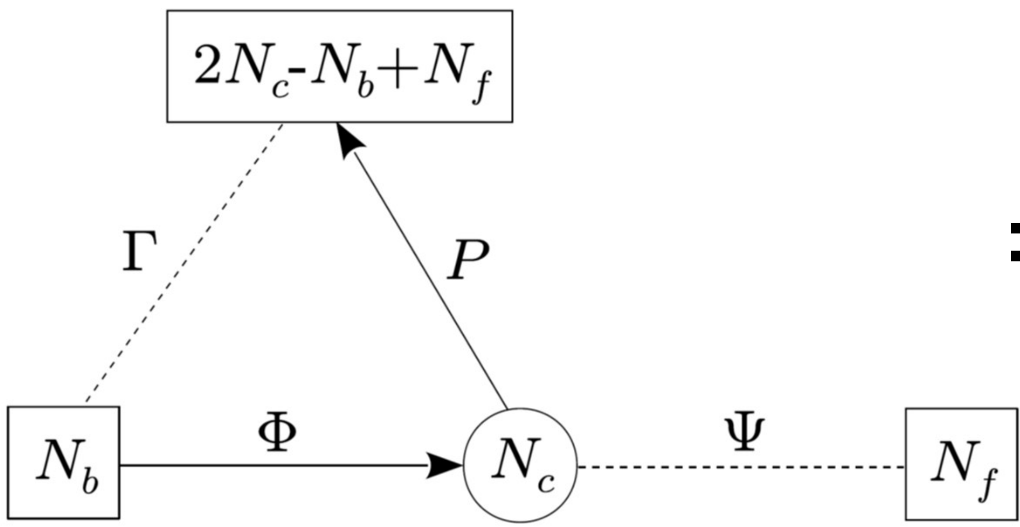
u
d
u


N_f
linear model
(SQED)
free fermions
free fermions
SUSY
SUSY
breaking
N_b
CP^{N-1}
free fermions
SUSY
breaking
N_c
0
1
N_b

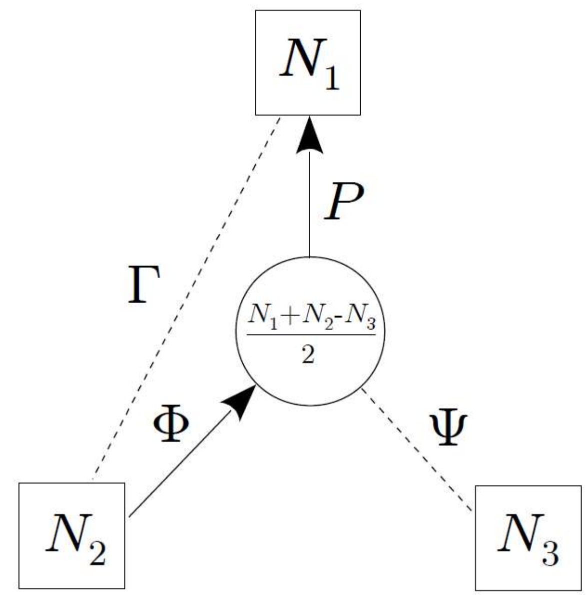

$2N_c$-N_b+N_f
Γ
P
N_b
Φ
N_c
Ψ
N_f


u
u
d

Hidden symmetry: Triality



=:



$$\mathcal{T}_{N_1,N_2,N_3} \cong \mathcal{T}_{N_3,N_1,N_2} \cong \mathcal{T}_{N_2,N_3,N_1}$$

$$Gr(k,N) \cong Gr(N-k,N)$$

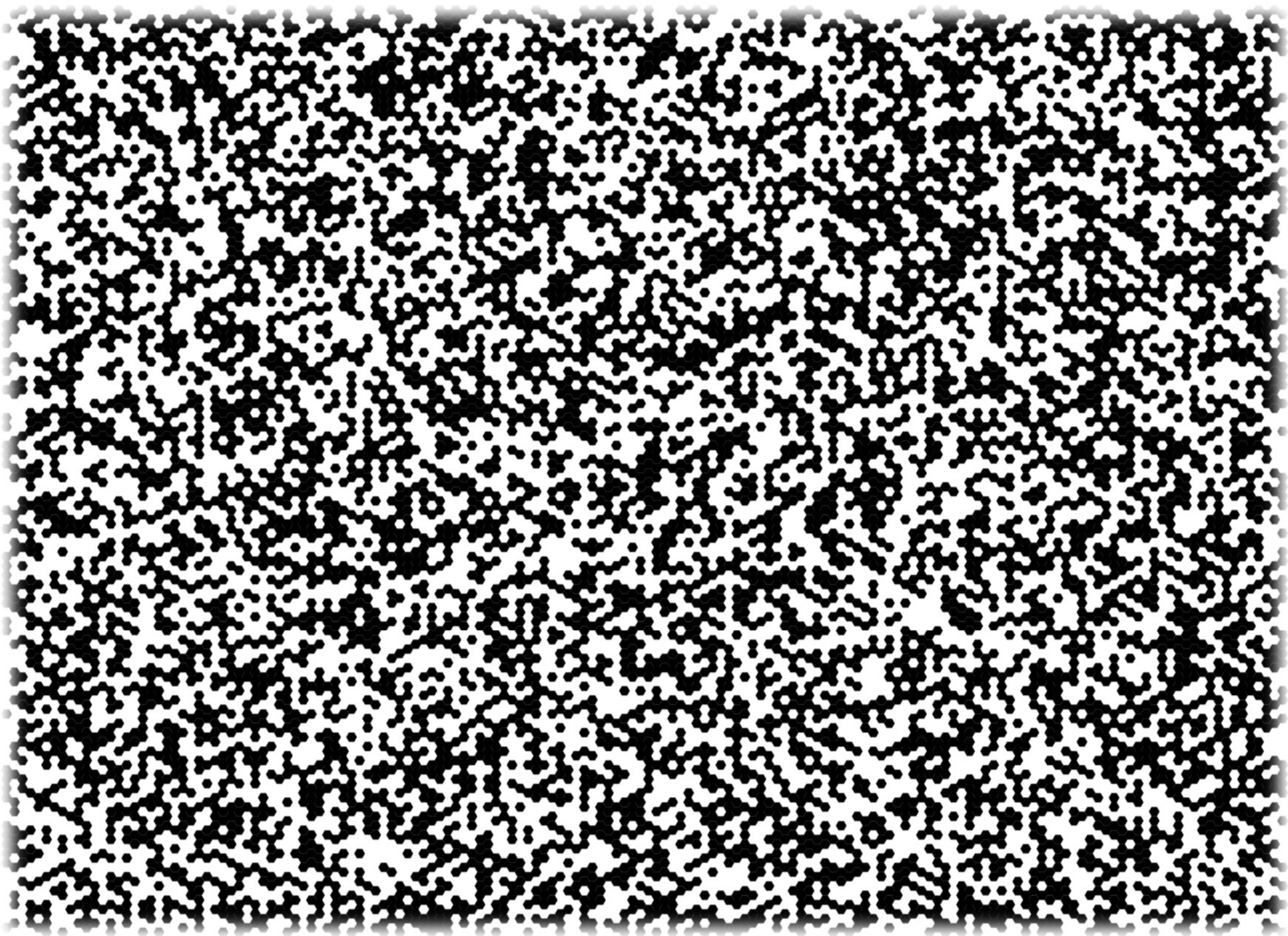
$$S \to Gr(k,N) \quad \cong \quad Q^* \to Gr(N-k,N)$$

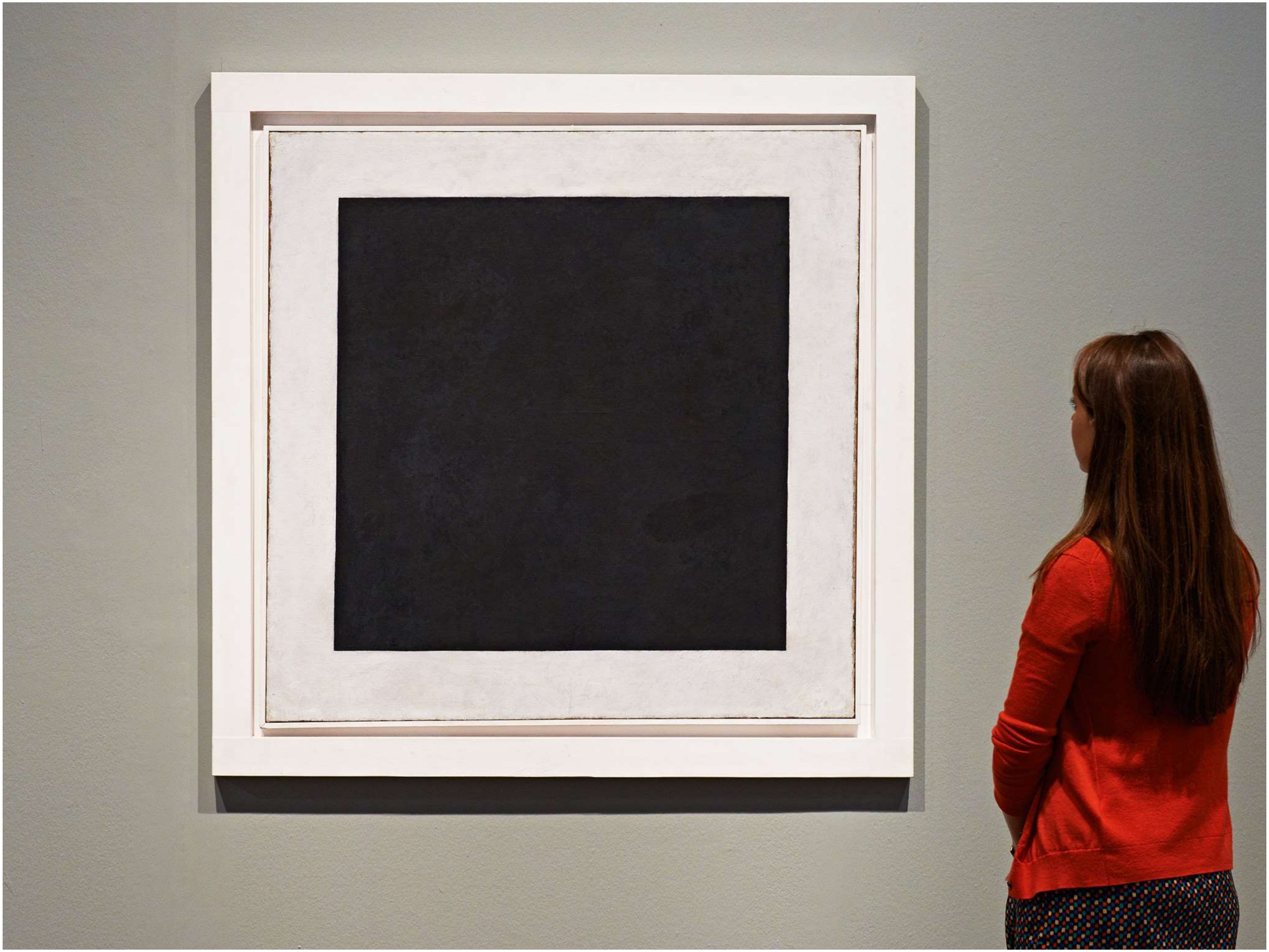
$$Q \to Gr(k,N) \quad \cong \quad S^* \to Gr(N-k,N)$$

where

$$0 \longrightarrow S \longrightarrow \mathcal{O}^N \longrightarrow Q \longrightarrow 0$$

Chiral de Rham $\left(\begin{array}{c} \Pi S^{\oplus N_3} \bigoplus \Pi Q^{\oplus N_2} \\ \downarrow \\ Gr\left(\frac{N_1+N_2-N_3}{2}, N_1\right) \end{array} \right)$



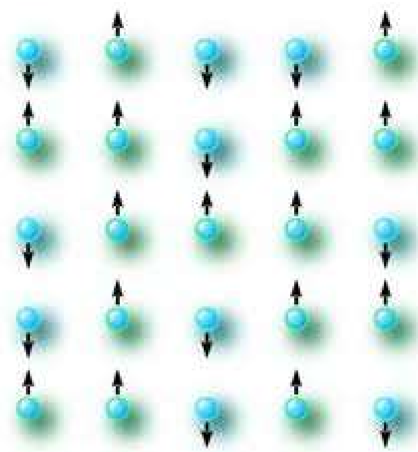


- Q-state Potts model (= Ising w/ spins: Q values)

$Q \to 1$ → Percolation (c=0)

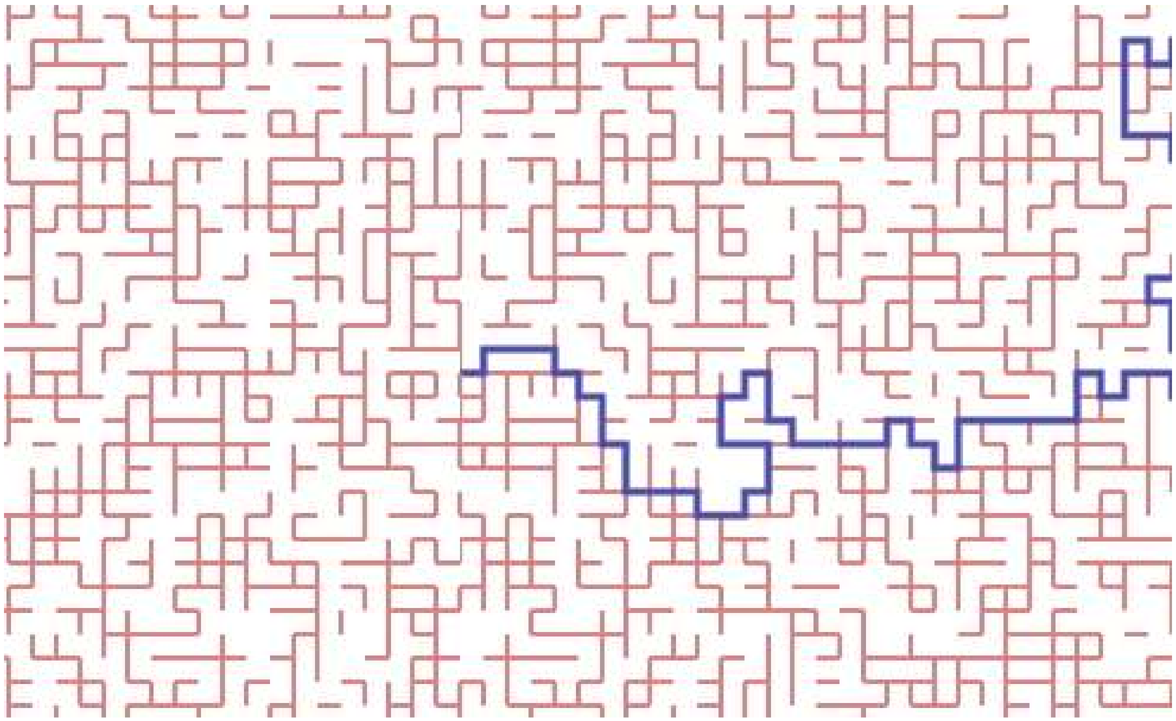
- O(n) model (another generalization of Ising)

$n \to 1$ → Ising



$n \to 0$ → self-avoiding walk

- QHE plateau phase transitions



John Cardy

$$C_{\text{perc}} = 1/8\sqrt{3}\pi \quad = \quad 0.0229720 \qquad \text{predicted}$$

$$= \quad 0.022972(1) \quad \text{measured}$$

(density of lops with area greater than A = C / A)

- Symplectic fermions:

$$c = -2d \qquad\qquad d \in \mathbb{Z}_+$$

- Triplet (1,p):

$$c = 13 - 6\left(p + \frac{1}{p}\right)$$

- Singlet (1,p):

$$c = 13 - 6\left(p + \frac{1}{p}\right)$$

affine W-algebra for ADE $\hookrightarrow$ Singlet $\hookrightarrow$ Triplet $\hookrightarrow$ Lattice VOA

B.Feigin, I.Tipunin (2010)

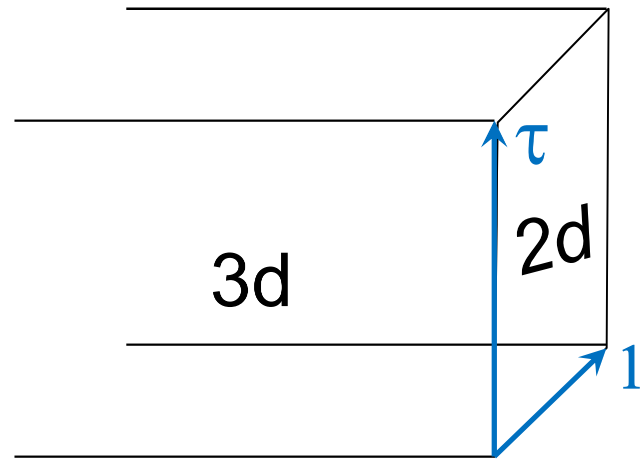
$$\text{Ker}_{\substack{short\\screening}} \begin{pmatrix} \text{Lattice} \\ \text{VOA} \end{pmatrix}$$

$$\|$$

$$\begin{matrix}\text{affine}\\ \text{W-algebra}\\ \text{for ADE}\end{matrix} \hookrightarrow \text{Singlet} \hookrightarrow \text{Triplet} \hookrightarrow \begin{matrix}\text{Lattice}\\ \text{VOA}\end{matrix}$$

$$\uparrow$$

$$\chi_{\text{singlet}} = \oint \frac{dx}{x} \chi_{\text{triplet}}$$



1) Half-index
= 2d/3d elliptic genus

A.Gadde, S.G, P.Putrov (2013)
Y.Yoshida, K.Sugiyama (2014)
N.Dorey, P.Zhao (2015)
S.G, D.Pei, C.Putrov, C.Vafa (2017)
T.Dimofte, D.Gaiotto, N.Paquette (2017)
S.G, B.Feigin (2018)
H.Jockers, P.Mayr (2019)
H.Jockers, P.Mayr, U.Ninad, A.Tabler (2019)
K.Costello, T.Dimofte, D.Gaiotto (2020)
F.Ferrari, P.Putrov (2020)
⋮

3d $\mathcal{N} = 2$ multiplet	boundary condition
$U(1)$ vector with $k = -1$ super-CS	Neumann
chiral with charge $+1$	Dirichlet
chiral with charge -1	Dirichlet

$$\int_{|x|=1} \frac{dx}{x} (x^{-1}q;q)_\infty (xq;q)_\infty \; = \; \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q;q)_n^2}$$

$$= \; 1 + q^2 + 2q^3 + 3q^4 + 4q^5 + 6q^6 + 8q^7 + \ldots$$

3d $\mathcal{N}=2$ multiplet	boundary condition
$U(1)$ vector with $k=-1$ super-CS	Neumann
chiral with charge $+1$	Dirichlet
chiral with charge -1	Dirichlet

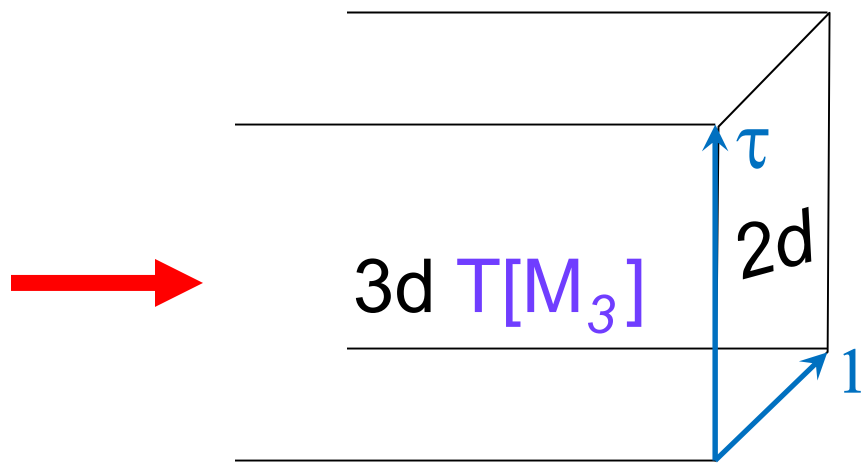
$$\int_{|x|=1} \frac{dx}{x} (x^{-1}q;q)_\infty (xq;q)_\infty = \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q;q)_n^2}$$

$$= \chi_{\text{log-VOA}}$$

c = - 2

M.Cheng, S.Chun, B.Feigin, F.Ferrari, S.G., S.Harrison

3-manifold



$$\widehat{Z}_a(M_3) \;=\; q^{\Delta}\left(c_0 + c_1 q + c_2 q^2 + \ldots\right) \qquad c_i \in \mathbb{Z}$$

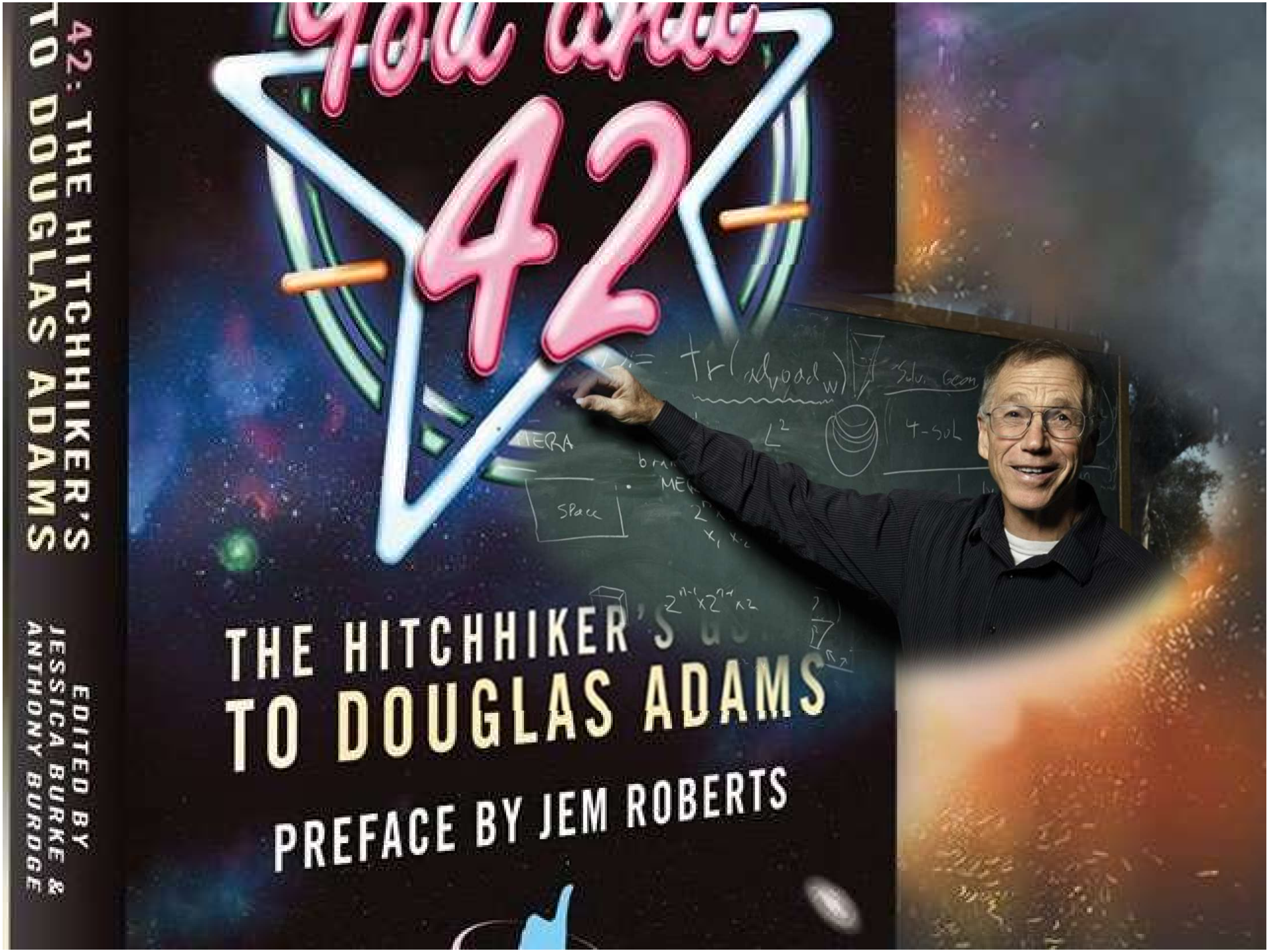
$M_3 = S^3_{-1}($  $) = S^3_{+1}($  $)$

"scaling dimension"

$$\widehat{Z}(q) = q^{1/2}(1 - q - q^5 + q^{10} - q^{11} + q^{18} + \cdots)$$

$$= q^{1/2} \sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n+1)}{2}}}{(q^{n+1};q)_n}$$

$=$ character of (1,p) "singlet" log-VOA with p = 42


42: THE HITCHHIKER'S
TO DOUGLAS ADAMS
EDITED BY
JESSICA BURKE &
ANTHONY BURDGE
42
THE HITCHHIKER'S
TO DOUGLAS ADAMS
PREFACE BY JEM ROBERTS

Conjecture ("3d Modularity"):

$$\widehat{Z}(q) = \chi_{\mathrm{VOA}[M_3]}$$

M.Cheng, S.Chun, F.Ferrari, S.G., S.Harrison (2018)

"scaling dimension"

$$\widehat{Z} = q^{\Delta} \left(a_0 + a_1 q + a_2 q^2 + \ldots \right) \quad \in \; q^{\Delta} \mathbb{Z}[[q]]$$

Conjecture ("mirror symmetry"):

$\widehat{Z}(M_3, q) = \chi(q)$ ⟷ $\widehat{Z}(-M_3, q) = \chi(q^{-1})$

Character of a chiral algebra

Character of a "mirror" chiral algebra

Topology	Physics	Algebra
		Modular Tensor Categories
3-manifolds →	3d N=2 →	
		(mock & quantum) modular forms
4-manifolds →	2d (0,2) SQCD →	
		VOA characters
		Topological Modular Forms